ARTICLE NO. 80-4126

Modal-Space Control of Large Flexible Spacecraft Possessing Ignorable Coordinates

L. Meirovitch* and H. Öz†
Virginia Polytechnic Institute and State University, Blacksburg, Va.

One problem peculiar to spacecraft in free space is that of ignorable coordinates, which are present when the spacecraft has no natural ability of preventing rigid-body motions. Ignorable coordinates can be eliminated from the problem formulation by introducing a reduced state vector. However, this results in having the ignorable coordinates uncontrolled. This paper solves this problem by introducing a dual-level control. The first-level controls are designed to control the ignorable coordinates and they lead to a positive definite "augmented system" and the second-level controls are designed for the final control of the complete state vector, including the ignorable coordinates. The second-level controls are designed in modal space and can be based on several types of control laws. In particular, proportional control providing artificial viscous damping, proportional optimal control, and nonlinear on-off control are discussed. A numerical example, illustrating all of these control laws, is presented.

I. Introduction

THE control of large flexible spacecraft has received a great deal of attention recently.[1] Strictly speaking, a flexible spacecraft represents a distributed parameter system. Practical considerations, however, dictate that the spacecraft be represented by a discrete system. References 2-13 are concerned with various aspects of the control of such discrete systems. For good simulation of a flexible spacecraft, the discretized model must be of relatively high order. References 9-13 present algorithms particularly suited to the control of high-order discrete systems associated with flexible spacecraft. Assessments of the methods presented in Refs. 1-13, as well as in other related investigations, are given in Refs. 14 and 15.

The control of large flexible spacecraft can be divided into three different but intimately related tasks. The first is the positional control, which can be characterized as the effort to maintain the spacecraft in the desired orbit. The second task is the attitude control, which is the effort to preserve a certain orientation of the spacecraft in space. The third task is the vibration control of the elastic members, which is frequently referred to as configuration or shape control. The three types of motion are coupled so that they must be treated simultaneously. The orbital and attitude motions are described by coordinates depending on time alone. On the other hand, the elastic vibrations are generally described by coordinates depending on time and space. Hence, the system equations of motions are hybrid, as they include both ordinary and partial differential equations. Hybrid sets of equations of the type associated with spacecraft generally do not admit closed-form solution, so that they are customarily transformed into infinite sets of ordinary differential equations by a procedure known as spatial discretization. For practical reasons, these infinite sets are truncated by retaining only a finite set of differential equations.[16] It should be clear, however, that the finite-dimensional discretized system represents only an approximation of the actual hybrid system.

A class of approaches to the control of discrete systems is generally referred to as modal control. As pointed out in Ref. 14, however, there are fundamental differences among modal control approaches. The approach advocated by these authors[9-13] was developed especially for high-order dynamical systems. It is based on a transformation from the actual space to the "modal space." The control laws are designed in the modal space, which permits independent control of each individual mode, so that the response of one mode does not affect the response of any other mode.

Modal-space control has been developed by these authors for the purpose of controlling flexible spacecraft exhibiting gyroscopic behavior, but the idea is not restricted to gyroscopic systems. Indeed, this paper presents an adaptation of the method to nongyroscopic systems. Because nongyroscopic and gyroscopic systems possess different dynamic characteristics, although the general idea of independent control in the modal space remains the same, the computational details are different.

The tasks of positional, attitude, and shape control are frequently treated separately. In fact, some of the tasks are often ignored entirely. Yet, the three tasks are coupled dynamically and must be treated simultaneously, as forces designed to control one type of motion are likely to excite other types of motion. In this paper, the three tasks are treated simultaneously.

One problem peculiar to spacecraft in free space is that of ignorable coordinates, which are present when the spacecraft has no natural ability of preventing rigid-body motions. This paper solves this problem by introducing a dual-level control. The first-level controls are designed to control the ignorable coordinates and they lead to a positive definite "augmented system." The second-level controls are designed for the final control of the complete state vector, including the ignorable coordinates, and any control spillover into the elastic coordinates resulting from the first-level controls. The modal-space control is implemented by using several types of control laws. In particular, proportional control providing artificial viscous damping, proportional optimal control, and nonlinear on-off control are discussed. A numerical example illustrating all of the preceding control laws is presented.

Presented at the Second VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, Va., June 21-23, 1979; submitted July 23, 1979; revision received Feb. 15, 1980.

Index categories: Spacecraft Dynamics and Control; Structural Dynamics.

*Reynolds Metals Professor, Dept. of Engineering Science and Mechanics. Associate Fellow AIAA.

†Assistant Professor, Dept. of Engineering Science and Mechanics. Member AIAA.

II. Lagrange's Equations of Motion

Let us consider a nongyroscopic flexible spacecraft and define a set of spacecraft axes $x_C y_C z_C$ with the origin at the spacecraft mass center C when the spacecraft is in the undeformed state. For convenience, we shall choose $x_C y_C z_C$ as the principal axes of the undeformed spacecraft. Assuming small angular motions of the reference frame $x_C y_C z_C$, the displacement vector of a nominal point in the spacecraft relative to an inertial space can be written in the form

$$u(P,t) = u_C(t) - \tilde{r}\theta(t) + u_E(P,t) \tag{1}$$

where $u_C(t)$ is the displacement vector of the center C, $\theta(t)$ is the angular displacement vector of the frame $x_C y_C z_C$, $u_E(P,t)$ is the elastic displacement of the nominal point P relative to the frame, and $\tilde{r}$ is a skew symmetric matrix representing the matrix counterpart of the vector operation $r\times$, in which r is the position vector of the nominal point P relative to the spacecraft axes. The absolute velocity vector is simply

$$\dot{u}(P,t) = \dot{u}_C(t) - \tilde{r}\dot{\theta}(t) + \dot{u}_E(P,t) \tag{2}$$

where dots represent derivatives with respect to time.

Using Eq. (2), the system kinetic energy is obtained in the general form

$$T = \tfrac{1}{2}\int_m \dot{u}^T\dot{u}\,dm = \tfrac{1}{2}\int_m [\dot{u}_C - \tilde{r}\dot{\theta} + \dot{u}_E]^T[\dot{u}_C - \tilde{r}\dot{\theta} + \dot{u}_E]\,dm$$

$$= \tfrac{1}{2}\dot{u}_C^T M_C \dot{u}_C + \tfrac{1}{2}\dot{\theta}^T J\dot{\theta} + \tfrac{1}{2}\int_m \dot{u}_E^T\dot{u}_E\,dm$$

$$+ \dot{u}_C^T\int_m \dot{u}_E\,dm + \dot{\theta}^T\int_m \tilde{r}\dot{u}_E\,dm \tag{3}$$

where $M_C = \text{diag}[m\ m\ m]$ is a mass matrix, in which $m = \int_m dm$ is the total mass of the spacecraft, and $J = \int_m \tilde{r}^T\tilde{r}\,dm = \text{diag}[A\ B\ C]$ is the mass moment of inertia matrix of the spacecraft in undeformed state about the spacecraft axes. We note that $\int_m \tilde{r}\,dm = 0$ by virtue of the fact that point C is the mass center.

The potential energy is due to elasticity alone and can be written in the symbolic form

$$V = \tfrac{1}{2}[u_E, u_E] \tag{4}$$

where $[u_E, u_E]$ is an energy inner product.[16]

Letting $f(P,t)$ be a distributed force and $m(P,t)$ a distributed torque, which include distributed control forces and torques, respectively, the virtual work is

$$\delta W = \int_D (f^T\delta u + m^T\delta\tfrac{1}{2}\nabla\times u)\,dD$$

$$= \int_D [f^T(\delta u_C - \tilde{r}\delta\theta + \delta u_E) + m^T(\delta\theta + \tfrac{1}{2}\nabla\times\delta u_E)]\,dD$$

$$= Q_{uC}^T\delta u_C + Q_\theta^T\delta\theta + \int_{D_E}\hat{Q}_{uE}^T\delta u_E\,dD_E \tag{5}$$

where

$$Q_{uC} = \int_D f\,dD \quad Q_\theta = \int_D (\tilde{r}f + m)\,dD \quad \hat{Q}_{uE} = f + \tfrac{1}{2}(\nabla\times)^T m \tag{6}$$

are generalized forces in which D is the domain of extension of the spacecraft and D_E that of the elastic member.

Using Eqs. (3-5), we obtain a hybrid set of Lagrange's equations of motion, as the equations for u_C and θ are ordinary differential equations and that for u_E is a partial differential equation. For brevity, we shall not write them explicitly, particularly since they do not lend themselves to closed-form solution.

It is common practice to discretize the partial differential equation for u_E by introducing the transformation

$$u_E(P,t) = \Phi(P)\zeta(t) \tag{7}$$

where $\Phi(P)$ is a $3\times n$ matrix of admissible functions, and $\zeta(t)$ is an n-vector of generalized elastic displacements. Introducing Eq. (7) into Eqs. (3-5), the discretized Lagrange's equations can be shown to be

$$M_C\ddot{u}_C + P_E\ddot{\zeta} = Q_{uC} \qquad J\ddot{\theta} + H_E\ddot{\zeta} = Q_\theta \tag{8a}$$

$$P_E^T\ddot{u}_C + H_E^T\ddot{\theta} + M_E\ddot{\zeta} + K_E\zeta = Q_\zeta \tag{8b}$$

where

$$M_E = \int_{m_E}\Phi^T\Phi\,dm_E \qquad P_E = \int_{m_E}\Phi\,dm_E$$

$$H_E = \int_{m_E}\tilde{r}\Phi\,dm_E \qquad K_E = [\Phi,\Phi] \tag{9}$$

$$Q_\zeta = \int_{D_E}[\Phi^T f + \tfrac{1}{2}(\nabla\times\Phi)^T m]\,dD_E \tag{10}$$

Equations (8) can be arranged in the matrix form

$$M\ddot{q} + Kq = Q \tag{11}$$

where

$$M = \begin{bmatrix} M_C & 0 & P_E \\ 0 & J & H_E \\ P_E^T & H_E^T & M_E \end{bmatrix} \quad K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_E \end{bmatrix} \tag{12a,b}$$

are coefficient matrices, which can be recognized as the mass and stiffness matrices, respectively, and $q = [u_C^T\ \theta^T\ \zeta^T]^T$ is the $(6+n)$-dimensional configuration vector. We note that the coordinates u_C and θ do not appear explicitly in the equations of motion, so that they can be regarded as ignorable. The mass matrix in Eqs. (12) is positive definite by definition, and the stiffness matrix is positive semidefinite.

III. State Equations

Equation (11) describe the motion of the system in the configuration space. Because the system is nongyroscopic, the equations can be used to control the system in the configuration space. However, much of the control theory works with the state space rather than the configuration space. This is particularly true of optimal control. In view of this, we wish to transform the Lagrange equations to state equations.

The state of the system and the associated force vector are defined as $x = [\dot{q}^T \mid q^T]^T$ and $X = [Q^T \mid 0^T]^T$, respectively. Moreover, introducing the coefficient matrices

$$I = \left[\begin{array}{c|c} M & 0 \\ \hline 0 & K \end{array}\right] \qquad G = \left[\begin{array}{c|c} 0 & K \\ \hline -K & 0 \end{array}\right] \tag{13}$$

where I is symmetric and G is skew symmetric, the state equations can be written in the form

$$I\dot{x} + Gx = X \tag{14}$$

Equation (14) has the same form as that of the gyroscopic system treated in Ref. 17, with the notable exception that the matrix I is not positive definite, but only positive semidefinite. Indeed, from Eq. (12b), we conclude that the equations have six zero rows corresponding to the ignorable coordinates

u_C, θ. Hence, while the displacement rates $\dot{u}_C$ and $\dot{\theta}$ are controllable, the displacements u_C and θ are uncontrollable. In the following we propose a method that permits the control of the complete state of the system, including the ignorable coordinates.

Let us return to Lagrange's equations, Eqs. (8a), and write the vectors Q_{uC} and Q_θ as

$$Q_{uC} = Q_{uC_1} + Q_{uC_2} \qquad Q_\theta = Q_{\theta_1} + Q_{\theta_2} \tag{15}$$

where Q_{uC_1} and Q_{θ_1} are assumed to have the special form

$$Q_{uC_1} = -K_{uC} u_C \qquad Q_{\theta_1} = -K_\theta \theta \tag{16}$$

in which K_{uC} and K_θ are 3×3 real symmetric positive definite matrices. In fact, they can be assumed to be diagonal with real positive elements. Introducing Eqs. (15) and (16) into Eqs. (8a) we can write the "augmented Lagrange equations" for u_C and θ

$$M_C \ddot{u}_C + P_E \ddot{\xi} + K_{uC} u_C = Q_{uC_2} \qquad J\ddot{\theta} + H_E \ddot{\xi} + K_\theta \theta = Q_{\theta_2} \tag{17}$$

This permits us to write the "augmented state equation"

$$I_a \dot{x} + G_a x = X_a \tag{18}$$

where

$$I_a = \left[\begin{array}{c|c} M & 0 \\ \hline 0 & K_a \end{array}\right] \qquad G_a = \left[\begin{array}{c|c} 0 & K_a \\ \hline -K_a & 0 \end{array}\right] \tag{19}$$

in which $K_a = \text{block-diag}\ [K_{uC} \quad K_\theta \quad K_E]$, and $X_a = [Q_{uC_2}^T Q_{\theta_2}^T Q_\xi^T | 0^T]^T$. This time, however, the matrix I_a is positive definite instead of being only positive semidefinite. Note that the magnitude of the elements of K_{uC} and K_θ can be chosen such that the lowest natural frequencies of the augmented system are sufficiently large so as to permit a desirable control system bandwidth.

The separation of the vectors Q_{uC} and Q_θ into two parts may appear like a mathematical artifice, but has in fact some physical meaning. Indeed, the vectors Q_{uC_1} and Q_{θ_1} can be regarded as representing the portion of the controls designed primarily to control the rigid-body motions, whereas the vectors Q_{uC_2}, Q_{θ_2}, and Q_ξ represent the controls designed to control the full state of the system, including any spillover that may result from the application of Q_{uC_1} and Q_{θ_1}.

IV. The Eigenvalue Problem of the Open-Loop Augmented System

For modal control, one must solve the eigenvalue problem associated with Eq. (18). The eigenvalue problem can be written in the form

$$\lambda_r \left[\begin{array}{c|c} M & 0 \\ \hline 0 & K_a \end{array}\right] x_r = \left[\begin{array}{c|c} 0 & -K_a \\ \hline K_a & 0 \end{array}\right] x_r \tag{20}$$

which has pure imaginary eigenvalues $\lambda_r = \pm i\omega_r$ $(r = 1, 2, \ldots, 6+n)$ and complex conjugate eigenvectors $x_r = y_r + iz_r$ and $x_r = y_r - iz_r$. The eigenvalue problem, (Eq. (20), is a special case of that for gyroscopic systems treated in Ref. 17, obtained by setting the gyroscopic matrix equal to zero. The eigenvectors y_r and z_r are orthogonal with respect to the matrix I_a, and in some sense with respect to the matrix G_a, and they can be normalized so as to satisfy[17]

$$X^T I_a X = 1 \qquad -X^T G_a X = \Lambda \tag{21a,b}$$

where X is the real modal matrix

$$X = [y_1 \ z_1 \ y_2 \ z_2 \ldots \ y_{6+n} \ z_{6+n}] \tag{22}$$

and

$$\Lambda = \text{block-diag}\ \Lambda_r = \text{block-diag} \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix} \qquad (r = 1, 2, \ldots, 6+n) \tag{23}$$

The eigenvalue problem, Eq. (20), is of order $2(6+n)$ and its solutions are complex. For nongyroscopic systems, however, the problem can be reduced to one of order $6+n$ only, and one whose solutions are real. To show this, let us introduce the notation $x_r = [x_{rU}^T | x_{rL}^T]^T$, where x_{rU} and x_{rL} are the upper and lower halves of the vector x_r, respectively. Then, the eigenvalue problem, Eq. (20), can be split into

$$\lambda_r M x_{rU} = -K_a x_{rL} \qquad \lambda_r K_a x_{rL} = K_a x_{rU} \tag{24a,b}$$

Recalling that $\lambda_r = i\omega_r$, Eq. (24b) yields $x_{rU} = i\omega_r x_{rL}$, so that we obtain the $(6+n) \times (6+n)$ eigenvalue problem

$$\omega_r^2 M x_{rL} = K_a x_{rL} \qquad (r = 1, \ldots, 6+n) \tag{25}$$

where x_{rL} are real. Of course, ω_r^2 are real. The solution of the eigenvalue problem of order $6+n$, Eq. (25), in conjunction with $x_{rU} = i\omega_r x_{rL}$, yields the solution of the eigenvalue problem of the order $2(6+n)$, Eq. (20). Indeed, the $2(6+n)$-dimensional eigenvector x_r has the form $x_r = [i\omega_r x_{rL}^T | x_{rL}^T]^T (r = 1, 2, \ldots, 6+n)$. To be consistent with Eqs. (21), it is easy to verify that the $(6+n)$-dimensional eigenvectors x_{rL} must satisfy the orthonormality relations

$$x_{rL}^T M x_{sL} = (1/\omega_r^2)\delta_{rs} \quad x_{rL}^T K_a x_{sL} = \delta_{rs} \quad (r, s = 1, 2, \ldots, 6+n) \tag{26}$$

Recalling that the real and imaginary parts of the eigenvector x_r were denoted by y_r and z_r, respectively, it follows that for a nongyroscopic system $y_r = [0^T | x_{rL}^T]^T$ and $z_r = [\omega_r x_{rL}^T | 0^T]^T$ $(r = 1, 2, \ldots, 6+n)$, so that the modal matrix, Eq. (22), takes the form

$$X = \begin{bmatrix} 0 & \omega_1 x_{1L} & 0 & \omega_2 x_{2L} & \ldots & 0 & \omega_{6+n} x_{6+n,L} \\ x_{1L} & 0 & x_{2L} & 0 & \ldots & x_{6+n,L} & 0 \end{bmatrix} \tag{27}$$

V. State Equations in Canonical Form

Let us now return to Eq. (18) and introduce the transformation

$$x = Xw \tag{28}$$

where X is the modal matrix given by Eq. (27). Inserting Eq. (28) into Eq. (18), multiplying on the left by X^T, and making use of the orthonormality relations, Eqs. (21), we obtain

$$\dot{w} = \Lambda w + W \tag{29}$$

where

$$W = X^T X_a \tag{30}$$

In view of the block-diagonal form of Λ, we conclude that Eq. (29) represents a set of $6+n$ modal equations of the form

$$\dot{w}_r = \Lambda_r w_r + W_r \qquad (r = 1, 2, \ldots, 6+n) \tag{31}$$

where

$$w_r = [\xi_r \ \eta_r]^T \qquad W_r = [W_{\xi r} \ W_{\eta r}]^T \qquad (r=1,2,...,6+n) \tag{32a,b}$$

are two-dimensional modal state and control vectors, respectively.

Next, let us introduce the notation

$$Q = Q_1 + Q_2 \tag{33}$$

where Q_1 and Q_2, referred to here as first- and second-level generalized controls, are given by

$$Q_1 = [Q_{uC_1}^T \ Q_{\theta_1}^T \ 0^T]^T \qquad Q_2 = [Q_{uC_2}^T \ Q_{\theta_2}^T \ Q_\zeta^T]^T \tag{34a,b}$$

Then, recognizing that

$$X_a = [Q_\zeta^T \mid 0^T]^T \tag{35}$$

and from Eqs. (21a) and (30) that

$$X_a = I_a X W \tag{36}$$

we conclude that

$$W_{\xi r} = 0 \qquad W_{\eta r} = \omega_r x_{rL}^T Q_2 \qquad (r=1,2,...,6+n) \tag{37}$$

For future reference, we wish to note the following relation

$$w = X^{-1} x = X^T I_a x \tag{38}$$

Considering Eq. (28) and the form of the modal matrix X, we deduce that the coordinates ξ_r and η_r are related to the displacements and velocities, respectively.

VI. The Control Problem

From Eqs. (34b) and (35) we conclude that the control vector X_a consists of three distinct parts, namely Q_{uC_2}, Q_{θ_2}, and Q_ζ. The first can be regarded as controlling the orbital motion u_C, the second the attitude motion θ, and the third the elastic motion ζ of the augmented system. The latter is sometimes referred to as shape control. It is clear from Eqs. (8b) and (17) that the three control efforts are coupled and that it is not possible to control one type of motion without exciting another type. Hence, all three control efforts must take place simultaneously.

The derivation of the generalized forces of Sec. II was based on distributed forces. Actuators, however, operate at discrete points. But, discrete forces can be treated as continuous by means of spatial Dirac delta functions. Indeed, denoting the force amplitude at point $P=P_i$ by $F_i(t)$, the distributed force can be written in the form

$$f(P,t) = F_i(t)\delta(P-P_i) \qquad (i=1,2,...,p^*) \tag{39}$$

where p^* denotes the number of forces. Similarly, if there are s^* concentrated torques with amplitudes $M_j(t)$ acting at point $P=P_j$, we can write

$$m(P,t) = M_j(t)\delta(P-P_j) \qquad (j=1,2,...,s^*) \tag{40}$$

Next, we wish to derive the relation between the generalized force vector Q_2 and the actual forces F_i and torques M_j. To this end, let us insert Eqs. (39) and (40) into Eqs. (6) and (10) and obtain

$$Q_{uC} = \int_D f \mathrm{d}D = \int_D F_i(t)\delta(P-P_i)\mathrm{d}D = \sum_{i=1}^{p^*} F_i(t)$$

$$\begin{aligned} Q_{\theta_2} &= \int_D (\tilde{r} f + m)\mathrm{d}D \\ &= \int_D [\tilde{r} F_i(t)\delta(P-P_i) + M_j(t)\delta(P-P_j)]\mathrm{d}D \\ &= \sum_{i=1}^{p^*} \tilde{r}(P_i) F_i(t) + \sum_{j=1}^{s^*} M_j(t) \\ Q_\zeta &= \int_{D_E} [\Phi^T f + \tfrac{1}{2}(\nabla\times\Phi)^T m]\mathrm{d}D_E \\ &= \int_{D_E} [\Phi^T F_i(t)\delta(P-P_i) \\ &\quad + \tfrac{1}{2}(\nabla\times\Phi)^T M_j(t)\delta(P-P_j)]\mathrm{d}D \\ &= \sum_{i=1}^{p^*} \Phi^T(P_i)F_i(t) + \tfrac{1}{2}\sum_{j=1}^{s^*} (\nabla\times\Phi)^T\Big|_{P=P_j} M_j(t) \end{aligned} \tag{41}$$

It will prove convenient to introduce the actual force vector $\mathfrak{F} = [M^T \mid F^T]^T$, where

$$\begin{aligned} M = [&M_{x1} \ M_{x2} \ldots M_{x(s_x+1)} \mid M_{y1} \ M_{y2} \ldots M_{y(s_y+1)} \\ &\mid M_{z1} \ M_{z2} \ldots M_{z(s_z+1)}]^T = [M_x^T \mid M_y^T \mid M_z^T]^T \end{aligned} \tag{42a}$$

$$\begin{aligned} F = [&F_{x1} \ F_{x2} \ldots F_{x(p_x+1)} \mid F_{y1} \ F_{y2} \ldots F_{y(p_y+1)} \\ &\mid F_{z1} \ F_{z2} \ldots F_{z(p_z+1)}]^T = [F_x^T \mid F_y^T \mid F_z]^T \end{aligned} \tag{42b}$$

in which $(s_x+1, \ s_y+1, \ s_z+1)$ and $(p_x+1, \ p_y+1, \ p_z+1)$ denote the number of components of torques and forces acting in the x, y, and z directions, respectively. Note that $s^* = 3+s_x+s_y+s_z = 3+s$, and $p^* = 3+p_x+p_y+p_z = 3+p$. We shall assume that one of each set of thrusters and torquers applying controls in a certain direction is located at the center of mass of the spacecraft, while the rest are distributed throughout the elastic domain of the spacecraft. These control forces and torques located at the center of mass are intended to have the maximum effect on the motion of the center of mass. This leaves s and p torquers and actuators to be located throughout the elastic domain of the spacecraft. In view of this, we can make the form of the vector $\mathfrak{F}_2(t)$ more explicit by rearranging in the form $\mathfrak{F}_2(t) = [F_C^T \mid M_C^T \mid M_E^T \mid F_E^T]^T$, where subscripts C and E refer to controls located at the center of mass and throughout the elastic domains, respectively, and

$$\begin{aligned} F_C &= [F_{Cx} \ F_{Cy} \ F_{Cz}]^T \qquad M_C = [M_{Cx} \ M_{Cy} \ M_{Cz}]^T \\ F_E &= [F_{x1} \ F_{x2} \ldots F_{xp_x} \mid F_{y1} \ F_{y2} \ldots F_{yp_y} \mid F_{z1} \ F_{z2} \ldots F_{zp_z}]_E^T \\ &= [F_{xp_x}^T \mid F_{yp_y}^T \mid F_{zp_z}^T]_E^T \end{aligned} \tag{43}$$

with M_E defined similarly. The control input vector $\mathfrak{F}_2$ is thus an N-vector, where $N = 6+s+p$. If some of the thrusters and torquers are capable of applying controls in more than one direction, then the total number N_A of the control devices will satisfy $N_A < N$. We shall assume that to each component of the input vector $\mathfrak{F}_2(t)$ there corresponds a thruster or a torquer, so that $N_A = N$. Hence, in the sequel, the number of thrusters and torquers will also denote the dimension of the input vector.

The relation between the generalized force vector Q_2 and the actual force vector can be written as $Q_2 = B\mathfrak{F}_2$, where B is a $(6+n)\times(6+s+p)$ matrix having the partitioned form

$$B=\begin{bmatrix} B_{uFC} & B_{uMC} & B_{uME} & B_{uFE} \\ B_{\theta FC} & B_{\theta MC} & B_{\theta ME} & B_{\theta FE} \\ B_{\zeta FC} & B_{\zeta MC} & B_{\zeta ME} & B_{\zeta FE} \end{bmatrix} \tag{44}$$

in which $B_{\theta FC}$ and B_{uMC} are 3×3 null matrices, B_{uME} is a $3\times s$ null matrix, and B_{uFE} and $B_{\theta FE}$ are the $3\times p$ matrices

$$B_{uFE}=\begin{bmatrix} \mathbf{1}^T_{p_x} & \mathbf{0}^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{1}^T_{p_y} & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{0}^T & \mathbf{1}^T_{p_z} \end{bmatrix} \quad B_{\theta FE}=\begin{bmatrix} \mathbf{0}^T & -z^T_{p_y} & y^T_{p_z} \\ z^T_{p_x} & \mathbf{0}^T & -x^T_{p_z} \\ -y^T_{p_x} & x^T_{p_y} & \mathbf{0}^T \end{bmatrix} \tag{45}$$

where $\mathbf{1}^T_{p_x}$, etc., denote p_x-dimensional vectors with all elements equal to unity, $x^T_{p_z}=[x(P_1)\ x(P_2)\ldots x(P_{p_z})]$, etc., and $B_{\theta ME}$ is a $3\times s$ matrix similar to B_{uFE} with p_x,p_y,p_z replaced by s_x,s_y,s_z. If the force F_C and the torque M_C are applied in a region for which there are no elastic deformations, then we conclude from Eqs. (5), (7), and (10) that the contributions of F_C and M_C to Q_ζ are zero. Hence, assuming that the spacecraft mass center lies in a rigid domain, such as a rigid platform, we note that $B_{\zeta FC}$ and $B_{\zeta MC}$ are $n\times 3$ null matrices. Moreover, $B_{\zeta ME}$ is the $n\times s$ matrix

$$B_{\zeta ME}=[B_{\zeta M_x} \mid B_{\zeta M_y} \mid B_{\zeta M_z}]_E \tag{46}$$

in which the submatrices are $n\times s_x$, $n\times s_y$, and $n\times s_z$, in that order, and

$$B_{\zeta M_x}=[\Phi_{z,y}(P_1)-\Phi_{y,z}(P_1)\ \Phi_{z,y}(P_2)-\Phi_{y,z}(P_2) \ldots \Phi_{z,y}(P_{s_x})-\Phi_{y,z}(P_{s_x})] \tag{47}$$

$B_{\zeta M_y}$ and $B_{\zeta M_z}$ are defined similarly with the cyclic permutation of the symbols x,y,z wherever they occur, and $\Phi_{z,y}=\partial\Phi_z/\partial y$, etc. Finally, $B_{\zeta FE}$ is the $n\times p$ matrix given by

$$B_{\zeta FE}=[B_{\zeta F_x} \mid B_{\zeta F_y} \mid B_{\zeta F_z}] \tag{48}$$

where the submatrices are $n\times p_x$, $n\times p_y$, and $n\times p_z$ matrices, respectively, and

$$B_{\zeta F_x}=[\Phi_x(P_1)\ \Phi_x(P_2)\ldots\Phi_x(P_{p_x})] \tag{49}$$

with $B_{\zeta F_y}$ and $B_{\zeta F_z}$ obtained from Eq. (49) by replacing x by y and z, respectively.

In the sequel, for simplicity, we shall assume that there are no torques applied throughout the elastic domain of the spacecraft, so that we can write

$$\mathfrak{F}_2(t)=[F^T_C \mid M^T_C \mid F^T_E] \qquad B=\begin{bmatrix} 1 & 0 & B_{uFE} \\ 0 & 1 & B_{\theta FE} \\ 0 & 0 & B_{\zeta FE} \end{bmatrix} \tag{50a,b}$$

where $\mathfrak{F}_2$ is a $(6+p)$-vector and B is a $(6+n)\times(6+p)$ matrix.

In the next section, we shall address the question of determining the control vector $\mathfrak{F}_2(t)$. The method of approach has been proposed by these authors for gyroscopic systems and is referred to as independent modal-space control.[9-13] We shall show that the method is also applicable to a nongyroscopic system, as represented by a spacecraft with no spinning parts.

VII. Independent Modal-Space Control

The basic idea of independent modal control is to design the controls first in the modal state space, i.e., to design the vector W such that the response of each pair of modal Eqs. (31) is not affected by the response of other modes. The control vector X_a on the coupled dynamics, Eq. (18), can then be synthesized from W by means of the transformation Eq. (36).

We shall assume that l pairs of modes of the system are to be controlled and that the modal equations are arranged such that the first l pairs of Eqs. (31) represent the modes to be controlled. Two types of independent modal control schemes, namely proportional and relay-type, on-off control, will be discussed. These schemes were presented by these authors earlier in Refs. 9, 10, and 13 for gyroscopic systems.

A. Nonoptimal Proportional Control

Proportional control can be regarded as artificial viscous damping. We shall take the modal control inputs W_r in the form

$$W_r=[0\ \ -2\zeta_r\omega_r\eta_r]^T \qquad (r=1,2,\ldots,l)$$
$$W_r=0 \qquad (r=l+1,\ldots,n+6) \tag{51}$$

so that the solution of Eqs. (31) in conjunction with Eqs. (51) can be shown to be decaying asymptotically.[9] The closed-loop eigenvalues are $-\zeta_r\omega_r \pm i\omega_{dr}$ $(r=1,2,\ldots,l)$, where $\omega_{dr}=\omega_r(1-\zeta_r^2)^{1/2}$. Equations (51) can be written in the more compact form

$$W=\begin{bmatrix} K_C & 0 \\ 0 & 0 \end{bmatrix} w=Kw \tag{52}$$

where

$$K_C=\text{diag}[0\ \ -2\zeta_1\omega_1\ \ 0\ \ -2\zeta_2\omega_2 \ldots 0\ \ -2\zeta_l\omega_l] \tag{53}$$

Using Eq. (36), the control vector Q_2 can be synthesized in the form

$$Q=MX_UW=-2\sum_{s=1}^{l}\zeta_s\omega_s^2 Mx_{sL}\eta_s \tag{54}$$

where X_U is the upper-half of the modal matrix X. Introducing the first equation from each pair in Eqs. (31) into Eq. (54), we obtain

$$Q=-2\sum_{s=1}^{l}\zeta_s\omega_s Mx_{sL}\dot{\xi}_s \tag{55}$$

It is not surprising to note that analogous results can be obtained without transforming to the state-vector formulation simply by considering the $(n+6)$-dimensional system

$$M\ddot{q}+K_aq=Q_2'=-2\sum_{s=1}^{l}\zeta_s\omega_s Mx_{sL}\dot{\xi}_s \tag{56}$$

B. Optimal Proportional Control

The control laws presented in the previous subsection were not optimal in any sense. However, for various reasons, it may be desirable to control the spacecraft while satisfying a certain performance measure. One such performance measure is known as quadratic performance measure. For control by modal synthesis, a quadratic modal performance measure J_r $(r=1,\ldots,l)$ can be defined for each mode independently of the performance measures for any other mode, leading to a performance measure of the form

$$J=\sum_{r=1}^{l} J_r=\sum_{r=1}^{l} \tfrac{1}{2}\{w_r^T(t_f)H_r w_r(t_f)+\int_0^{t_f}[w_r^T(t)Q_r w_r(t)+W_r^T R_r W_r]\mathrm{d}t\} \quad (57)$$

where w_r satisfies Eqs. (31). The final time t_f is assumed to be fixed, the matrices H_r and Q_r are positive semidefinite, and R_r is positive definite. Choosing the matrix Q_r as a 2×2 unit matrix, the first term in the integrand can be interpreted as the energy in the first l modes of the system, and the second term corresponds to the control effort. The term associated with the matrix H_r represents a penalty on the distance from the origin of the modal space at the end of the control period. Because each J_r is independent of the others,

$$\min J=\sum_{r=1}^{l}\min J_r$$

Assuming that the controls are not bounded and that $w_r(t_f)$ is free, and denoting optimal quantities by an asterisk, the optimal control law has the form[13]

$$W_r^*(t)=-R_r^{-1}E_r(t)w_r^*(t)=f_r^*(t)w_r^* \quad (r=1,\ldots,l) \quad (58)$$

where $E_r(t)$ satisfies the differential equation

$$\dot{E}_r(t)=-E_r\Lambda_r-\Lambda_r^T E_r-Q_r+E_r R_r^{-1}E_r \qquad E_r(t_f)=H_r \quad (59)$$

Equation (59) is a 2×2 matrix Riccati equation for each mode, and its solution can be obtained easily. The resulting optimal gain matrix f_r^* is time-dependent. Because $W_{\xi r}$ must be zero, according to the second of Eqs. (32b), the weighting matrix R_r will be assumed to be of the form $R_r=\mathrm{diag}[\infty\ R_{r2}]$. Then, from Eqs. (58), it follows that the first row of the matrix f_r^* is zero. The steady-state solution of Eq. (59), defined by $\dot{E}_r(t)=0$, is of special interest because it leads to a constant gain matrix. It is shown in Ref. 13 that the steady-state solution is

$$E_{12}=E_{21}=-\omega_r R_{r2}+\sqrt{\omega_r^2 R_{r2}^2+R_{r2}}$$
$$E_{22}=(R_r-2\omega_r^2 R_{r2}^2+2\omega_r R_{r2}\sqrt{\omega_r^2 R_{r2}^2+R_{r2}})^{1/2}$$
$$E_{11}=[\omega_r^{-2}+2\omega_r^{-1}R_r^{-1}(\omega_r^2 R_{r2}^2+R_{r2})^{3/2}-2R_{r2}^2\omega_r^2-R_{r2}]^{1/2} \quad (60)$$

The transient solution of Eq. (59) is also given in Ref. 13 but, for brevity, will not be presented here. The optimal modal control law for the steady-state case has the form

$$W_{\eta r}^*=[f_{r21}^*\ f_{r22}^*]w_r^*$$
$$=[\omega_r-\sqrt{\omega_r^2+R_{r2}^{-1}}-\{2\omega_r(-\omega_r+\sqrt{\omega_r^2+R_{r2}^{-1}})+R_{r2}^{-1}\}^{1/2}]\times\begin{bmatrix}\xi_r^*\\ \eta_r^*\end{bmatrix} \quad (61)$$

which also uses the coordinate ξ_r as feedback. Hence, the optimal control law uses feedback of both displacements and velocities, and it provides stiffening of the modes as well as damping. The optimal control law W^* in the matrix form can again be written as in Eq. (52), but K_C this time will have the form

$$K_C=\text{block-diag}\begin{bmatrix}0 & 0\\ f_{r21}^* & f_{r22}^*\end{bmatrix} \quad (r=1,\ldots,l) \quad (62)$$

The advantage of the performance measure in modal form, as given by Eq. (57), is that it leads to second-order Riccati equations, independently of the order of the system. This is a significant advantage over coupled control methods which yield Riccati equations equal in order to the order of the dynamical system to be controlled. Solutions of the Riccati equation of high-order suffer from problems of accuracy and computational time.

C. On-Off Control

Proportional control has the disadvantage that it must operate continuously. A scheme not suffering from this drawback is on-off control with deadband. This implies that some low-amplitude oscillations can be tolerated. In this case, the generalized control vector has the form

$$Q_2=\sum_{s=1}^{l}u_s M x_{sL} \quad (63)$$

where u_s are nonlinear functions of $\eta_s(t)$ given explicitly by

$$u_s=\{-k_s,\eta_s>d_s;\ 0,\ |\eta_s|\le d_s;\ k_s,\eta_s<-d_s\} \quad (64)$$

where d_s is a constant defining the deadband region. Introducing Eq. (63) into the second of Eqs. (37), we obtain

$$W_r=[0\ \ u_r/\omega_r]^T \quad (65)$$

The solution of Eqs. (31) has been obtained in Ref. 9 for each of the three intervals described in Eq. (64).

An application of on-off control to a gyroscopic system is presented in Ref. 9, which also includes a phase-plane plot for modal response. Noteworthy in that plot is the presence of chattering, a phenomenon that can be expected in on-off control.

Next, let us separate the controlled and uncontrolled modes in Eq. (29) so that $w=[w_C^T \mid w_R^T]^T$, where w_C and w_R denote the controlled and uncontrolled (residual) modes, respectively. Similarly, writing $\Lambda=\text{block-diag}\ [\Lambda_C \mid \Lambda_R]$ and using Eq. (52), one has for proportional control

$$\begin{bmatrix}\dot{w}_C\\ \hline \dot{w}_R\end{bmatrix}=\left[\begin{array}{c|c}\Lambda_C+K_C & 0\\ \hline 0 & \Lambda_R\end{array}\right]\begin{bmatrix}w_C\\ \hline w_R\end{bmatrix} \quad (66)$$

so that the *independent modal-space control scheme for nongyroscopic systems has the advantage that it causes no control spillover into the modeled uncontrolled modes of a discretized dynamical system.*

Finally, the actual control force vector $\mathfrak{F}_2(t)$ can be obtained from Q_2 through the inverse transformation $\mathfrak{F}_2=B^{-1}Q_2$, which requires that B^{-1} exist. Recalling the form of the matrix B, Eq. (50a), and the form of the vector Q_2, Eq. (34b), it can be shown that

$$\mathfrak{F}_2(t)=\left[\begin{array}{cc|c}1 & 0 & \\ & & -DB_{\zeta FE}^{-1}\\ 0 & 1 & \\ \hline 0 & 0 & B_{\zeta FE}^{-1}\end{array}\right]\begin{bmatrix}Q_{uC2}\\ Q_{\theta 2}\\ \hline Q_\zeta\end{bmatrix}=H_2Q_2 \quad (67a)$$

$$D=\begin{bmatrix}B_{uFE}\\ \hline B_{\theta FE}\end{bmatrix} \quad (67b)$$

The modal control laws can thus be realized uniquely if and only if the matrix $B_{\zeta FE}$ is a nonsingular square matrix which implies that the number of actuators located in the elastic domain of the spacecraft must be equal to the number of elastic generalized coordinates. Considering

Eqs. (35), (36), (52), and (67a), the actual control input for proportional control has the linear state feedback form $\mathfrak{F}_2(t) = [H_2 \,|\, 0]\; I_a X K w$, in which for nonoptimal proportional control K is the diagonal gain matrix given by Eqs. (52) and (53), having only l nonzero diagonal elements, and for optimal control K is the block-diagonal gain matrix given by Eqs. (52) and (62), which has only $2l$ nonzero elements. On the other hand, for on-off control the vector $\mathfrak{F}_2(t)$ has the form

$$\mathfrak{F}_2(t) = [H_2 \,|\, 0] I_a X [0 \;\; u_1/\omega_1 \;\; 0 \;\; u_2/\omega_2 \ldots u_l/\omega_l \;\; 0 \;\; 0 \ldots 0]^T \tag{68}$$

Due to the nature of the inputs, $u_1,\ldots,u_l$, as defined by Eq. (64), $\mathfrak{F}_2(t)$ will have the form of a staircase, representing a linear combination of on-off functions. In the case of nonlinear on-off control, even though a closed-loop relation of the form of Eq. (66) cannot be written, the stability characteristics of the modes of the system subjected to the nonlinear staircase type $\mathfrak{F}_2$ are known from the nature of the solutions. In particular, while controlled modes are damped out, the residual modes remain critically stable and no control spillover into the residual modes exists. It must be emphasized that without the decoupling procedure used here, the design of a nonlinear control force $\mathfrak{F}_2$ for coupled systems of high-order is intractable.[18] Furthermore, even if a stabilizing nonlinear $\mathfrak{F}_2$ can be designed for certain modes, the stability of the residual modes will in all likelihood be difficult to prove.

Note that the preceding developments assume that the full modal state vector is available through measurements, observer estimations, or a combination of both. The modal coordinates $w_r\,(r=1,\ldots,l)$ can indeed be obtained by means of a modal observer[12] without observation spillover from the coordinates $w_r\,(r=l+1,\ldots,n+6)$. The results of Ref. 12 are also verified by Balas[19] by a different approach to the observation spillover problem. Although Ref. 12 presents its results for a gyroscopic system, the modal observer design can be extended to the special case of nongyroscopic systems discussed here.

VIII. Numerical Example

As an illustration, a nonspinning flexible spacecraft was considered (Fig. 1). The same configuration with a uniformly spinning rotor, and hence representing a gyroscopic system, was studied in Refs. 10, 12, and 13. The system represents a nongyroscopic system, possessing six rigid-body degrees of freedom in the form of three translational and three rotational degrees of freedom. The equations of motion of the discretized system have the form of Eqs. (11) and (12). The following parameters were used: $m = 15{,}000.0$ kg, $A = 250.0$ kg-m^2, $B = 800.0$ kg-m^2, $C = 1{,}200$ kg-m^2

The elastic appendages were first modeled by the finite-element technique. Then, the motion of the appendages was simulated by using the cantilevered appendage modes as admissible functions. In the expansion of Eq. (7), six admissible functions were considered for each of the appendages so that the total number of elastic degrees of freedom was twelve. The vector ζ in Eq. (7) is therefore a 12-dimensional vector, the first six components representing the first appendage and the last six components representing the second appendage coordinates. The six elastic coordinates for each appendage correspond to the first out-of-plane bending, first in-plane bending, first torsional, second out-of-plane bending, second in-plane bending, and second torsional modes. Because the cantilever appendage modes were used as assumed functions, the stiffness matrix K_E is diagonal. Hence, the augmented stiffness matrix K_a is diagonal. The full state vector of the spacecraft is 36-dimensional.

The system dynamics was augmented (Sec. II) by introducing

$$K_{uC} = \text{diag}[1.0 \;\; 1.2 \;\; 1.5] \times 10^4 \text{N/m}$$

$$K_\theta = \text{diag}[5.0 \;\; 5.0 \;\; 5.0] \times 10^2 \text{N-m}$$

Upon formation of the matrices M and K_a, the 18th-order eigenvalue problem of Eq. (25) was solved by the Jacobi method. The modal matrix X was formed according to Eq. (27) by using the eigenvectors of Eq. (25). For brevity, we shall omit the eigensolution. Because in an eigensolution only the lower estimated natural frequencies are reliable estimates of the true natural frequencies and the higher natural frequencies are poor estimates of the true natural frequencies, an attempt was made to control only the modes corresponding to the first nine natural frequencies ($l=9$), thus leading to 18 controlled modes and 18 residual modes in the control problem formulation [Eq. (66)]. The pairs of modes to be controlled represent the first antisymmetric in-plane bending, first antisymmetric out-of-plane bending, first antisymmetric bending in the x_C-direction, first symmetric in-plane bending, first symmetric out-of-plane bending, first antisymmetric torsion, second symmetric out-of-plane bending, second symmetric in-plane bending, and second antisymmetric in-plane bending motions of the spacecraft. The spacecraft was subjected to initial velocities, but to no initial displacements.

The control of the spacecraft was achieved by three force actuators and three torquers located at the spacecraft center of mass and applying forces along and torques about the spacecraft axes $x_C y_C z_C$. In addition to these, six force actuators on each of the appendages were located antisymmetrically for a total of 18 actuators. The location of actuators on the elastic appendages is shown in Fig. 1. Actuators applying forces in the z_C-direction were located at points 1-4, and actuators applying forces in the y_C-direction were located at points 3 and 4 on each appendage.

The actual spacecraft is acted upon by the first-level controls applied at the mass center having the form $\mathfrak{F}_1 = Q_1 = -K_{uC} u_C - K_\theta u_\theta$. This process leads to the augmented spacecraft. The second-level control input vector on the augmented spacecraft is 18-dimensional and has the form

$$\mathfrak{F}_2 = [F_{Cx} \; F_{Cy} \; F_{Cz} \,|\, M_{Cx} \; M_{Cy} \; M_{Cz} \,|\, F_{E1} \; F_{E2} \ldots F_{E12}]^T$$

in which the pairs F_{E1}, F_{E2} and F_{E7}, F_{E8} represent two pairs of actuators applying forces in the y_C-direction on appendages 1 and 2, respectively (Fig. 1). Likewise, F_{E3}, F_{E4}, F_{E5}, F_{E6} and F_{E9}, F_{E10}, F_{E11}, F_{E12} represent four actuators each, eight in total, applying forces in the z_C-direction on appendages 1 and 2, respectively. With the choice of the locations for the torquers and force actuators, the matrix B of Eq. (50b) is computed to obtain the actual control input vector $\mathfrak{F}_2(t)$ from the independent modal control inputs $W(t)$, according to Eq. (36). The total control input vector acting on the spacecraft is the sum of the first-level controls and the second-level controls. With the choice of matrices K_{uC} and K_θ, the first-level control input has been already determined so that the main control design effort is the determination of the second-level controls $\mathfrak{F}_2$. Therefore, in the following, the second-level control design is presented. Block-diagrams of the control scheme are shown in Figs. 2 and 3.

The following parameters were used in the design of the modal control laws discussed in Sec. VII.

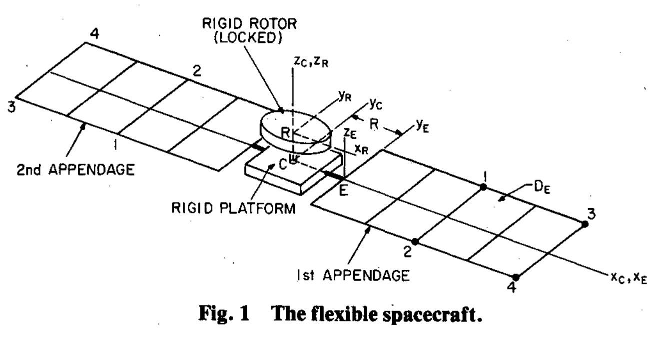

Fig. 1 The flexible spacecraft.

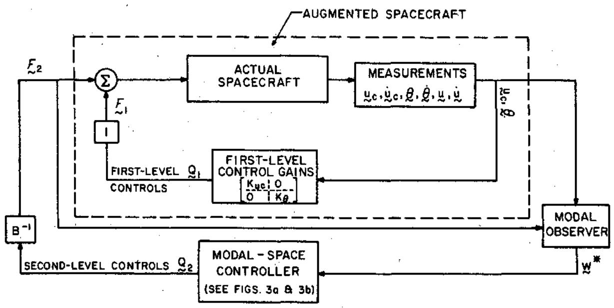

Fig. 2 Block-diagram for dual-level control.

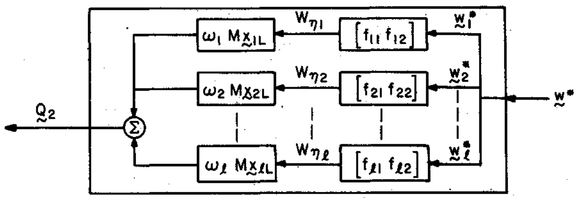

Fig. 3 a) Proportional modal-space control.

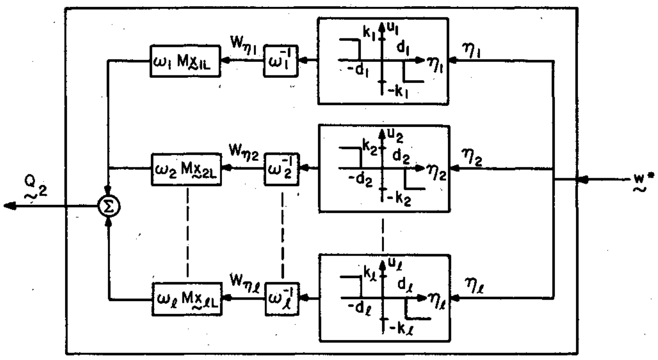

Fig. 3 b) On-off modal-space control.

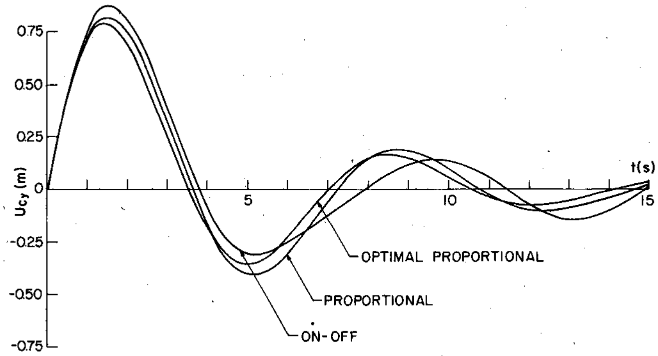

Fig. 4 a) Translation of the mass center in the y_C direction.

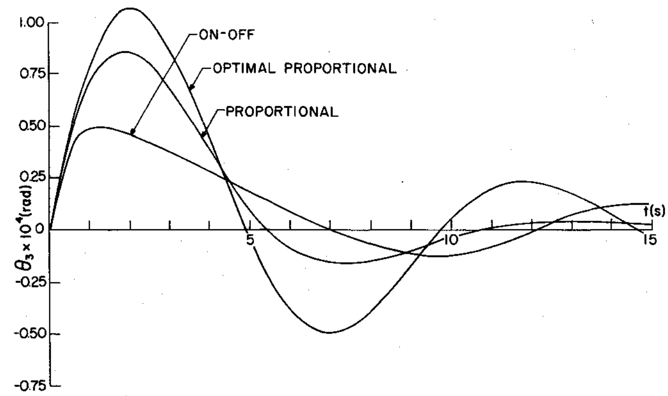

Fig. 4 b) Rotation of the spacecraft about z_C.

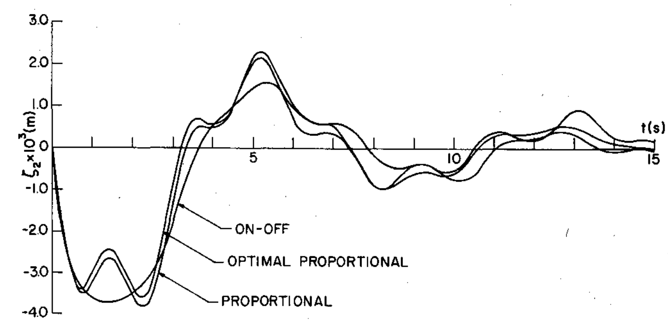

Fig. 4 c) First in-plane elastic coordinate of the first appendage.

A. Nonoptimal Proportional Control

The gain matrix K_C was chosen as

$$K_C = \text{diag}[0\ \ -0.6\ \ 0\ \ -0.4\ \ 0\ \ -0.4\ \ 0\ \ -0.4\ \ 0\ \ -0.4\ \ 0\ \ -0.4\ \ 0\ \ -0.4\ \ 0\ \ -0.4\ \ 0\ \ -0.4]$$

B. Optimal Proportional Control

The modal control input weighting sequence was

$$R_{12}=20.0,\ R_{22}=20.0,\ R_{32}=10.0,\ R_{42}=10.0,\ R_{52}=10.0,$$
$$R_{62}=15.0,\ R_{72}=10.0,\ R_{82}=15.0,\ R_{92}=20.0$$

yielding the nonzero elements f^*_{r21} and f^*_{r22} $(r=1,...,9)$ of the matrix K_C as

$$[f^*_{121}\ f^*_{122}\cdots f^*_{921}\ f^*_{922}] = [-0.03764\ \ -0.31398\ \vdots\ -0.03276\ \ -0.31453\ \vdots\ -0.05910\ \ -0.44330\ \vdots\ -0.05246\ \ -0.44391$$
$$-0.04881\ \ -0.44454\ \vdots\ -0.02351\ \ -0.36439\ \vdots\ -0.02479\ \ -0.44653\ \vdots\ -0.01012\ \ -0.36501\ \vdots\ -0.00754\ \ -0.31614]$$

C. On-Off Nonlinear Control

The deadband constants were

$$d_1=d_2=d_6=0.001,\ d_3=d_4=d_5=30.0,$$
$$d_7=0.3,\ d_8=0.03,\ d_9=0.005$$

and the modal control input constants were

$$k_1=0.015\omega_1^2,\quad k_2=0.007\omega_2^2,\quad k_3=75.0\omega_3^2,$$
$$k_4=100.0\omega_4^2,\quad k_5=100.0\omega_5^2,\quad k_6=0.005\omega_6^2,$$
$$k_7=1.00\omega_7^2,\quad k_8=0.07\omega_8^2,\quad k_9=0.0005\omega_9^2$$

A comparison of the coordinates u_{Cy}, θ_3, and ζ_2 associated with the in-plane motion can be made for each type of control by means of Figs. 4a-4c. It should be noted from these plots that the residual modes do not have any adverse effect, as one should expect, because the designed independent mode control laws do not permit their excitation. A comparison of the nonoptimal proportional control and optimal proportional control was made on the basis of the closed-loop

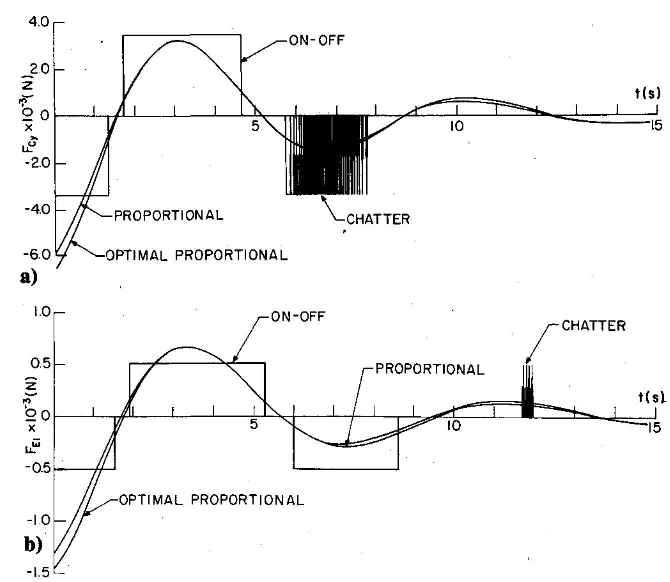

Fig. 5 Actual second-level control force in the y_C direction applied at a) the mass center and b) point 3 of the first appendage.

eigenvalues (not shown here for brevity). The most significant difference was noticed in the first mode (in-plane antisymmetric), for which the nonoptimal control provides faster damping. This effect is clearly seen in the plot of θ_3 vs t. No significant differences in the other coordinates are observed either, as the rest of the closed-loop poles for both types of proportional control do not differ appreciably. The corresponding diagonal elements of the matrix K_C are smaller for the optimal proportional control than for the nonoptimal control. However, by design, the optimal proportional control has a nonzero off-diagonal gain for each block. It should be pointed out that the efficiency of the control effort depends on the choice of the diagonal elements of K_C for nonoptimal control and on the weighting sequence R_{r2} ($r=1,...,9$) for the optimal control. Because there is a large measure of arbitrariness in this choice, either one of the two control schemes can be made superior to the other. It should be observed from Figs. 4a-4c that the on-off control scheme can be successfully implemented to obtain desired responses, and hence, effective controls. Indeed, the results compare favorably with those produced by proportional controls. The effectiveness of the on-off control scheme is even more vividly illustrated in Figs. 5a and 5b, which show the actual force input time histories for the force F_{Cy} applied at the spacecraft mass center, and for the force F_{EI} applied by the actuator located at position 3 on the first appendage tip in the y_C-direction. The comparative simplicity of the operation of the on-off controllers, with no compromise in the system response, can make them quite attractive for certain applications. It should also be observed from the Figs. 5a and 5b that the areas under the force diagrams, which represent the total impulses required for control, are practically the same as for the proportional controls, thus suggesting that equal amounts of control efforts are spent for both nonoptimal and optimal controls. Again, for the example presented here, the on-off control scheme compares favorably with the proportional controls.

IX. Conclusions

An independent modal-space control scheme for the control of positional, attitude, and elastic motions for a discretized model of a distributed-parameter flexible spacecraft with ignorable coordinates is presented. The control scheme consists of a dual-level control. The first-level control is designed to restrain the ignorable coordinates associated with the rigid-body motions. The net result is the creation of a positive definite "augmented system." The second-level control is designed in the modal space to control the full state of the augmented spacecraft, including the ignorable coordinates. The proposed independent modal-space control scheme is demonstrated for nonoptimal and optimal proportional controls, as well as for nonlinear, on-off control. The approach developed here is particularly suitable for high-order systems and is free of control spillover into the uncontrolled (residual) modes of the truncated dynamical model considered. Because the truncated model can be of very high dimension, unmodeled residual modes have no practical significance. A numerical example of a flexible dual-spin spacecraft with a locked rotor is presented.

References

[1]Meirovitch, L. (ed.), *Proceedings of the First VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft,"* Blacksburg, Va., June 1977.

[2]Poelaert, D.H., "A Guideline for the Analysis and Synthesis of Nonrigid-Spacecraft Control System," *ESA/ASE Scientific and Technical Review,* 1975, pp. 203-218.

[3]Balas, M., "Active Control of Flexible Systems," *Journal of Optimization Theory and Applications,* Vol. 25, No. 3, 1978, pp. 415-436.

[4]Balas, M., "Modal Control of Certain Flexible Dynamic Systems," *SIAM Journal on Control and Optimization,* Vol. 16, 1978, pp. 450-462.

[5]Balas, M., "Reduced-Order Control of Large Structures in Space," Paper 79-0196, 17th Aerospace Sciences Meeting, New Orleans, La., Jan. 1979.

[6]Skelton, R., "On the Minimal Controller Problem," *Proceedings of the First VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft,* edited by L. Meirovitch, Blacksburg, Va., June 1977, pp. 119-128.

[7]Skelton, R. and Likins, P.W., "On the Use of Model Error Systems in the Control of Large Scale Linearized Systems," *IFAC Symposium on Large Scale Systems, Theory, and Applications,* Udine, Italy, 1976.

[8]Skelton, R. and Likins, P.W., "Orthogonal Filters for Model Error Compensation in the Control of Nonrigid Spacecraft," *Journal of Guidance and Control,* Vol. 1, Jan.-Feb. 1978, pp. 41-49.

[9]Meirovitch, L., VanLandingham, H.F., and Öz, H., "Control of Spinning Flexible Spacecraft by Modal Synthesis," *Acta Astronautica,* Vol. 4, 1977, pp. 985-1010.

[10]Meirovitch, L. and Öz, H., "Observer Modal Control of Dual-Spin Flexible Spacecraft," *Journal of Guidance and Control,* Vol. 2, March-April 1979, pp. 101-110.

[11]Meirovitch, L., VanLandingham, H.F., and Öz, H., "Distributed Control of Spinning Flexible Spacecraft," *Proceedings of the First VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft,* edited by L. Meirovitch, Blacksburg, Va., June 1977, pp. 249-269; also *Journal of Guidance and Control,* Vol. 2, Sept.-Oct. 1979, pp. 407-415.

[12]Meirovitch, L. and Öz, H., "Modal-Space Control of Distributed Gyroscopic Systems," Paper 78-1421, AIAA/AAS Astrodynamics Conference, Palo Alto, Calif., Aug. 7-9, 1978; also *Journal of Guidance and Control,* Vol. 3, March-April 1980, pp. 140-150.

[13]Öz, H. and Meirovitch, L., "Optimal Modal-Space Control of Flexible Gyroscopic Systems," Paper 78-103, IAF, XXIXth Congress, Dubrovnik, Yugoslavia, Oct. 1978; also *Journal of Guidance and Control,* May-June 1980, pp. 218-226.

[14]Meirovitch, L. and Öz, H., "An Assessment of Methods for the Control of Large Space Structures," *Proceedings of the 1979 Joint Automatic Controls Conference,* Denver, Col., June 18-20, 1979, pp. 34-41.

[15]Balas, M., "Some Trends in Large Space Structure Control Theory: Fondest Hopes; Wildest Dreams," *Proceedings of the 1979 Joint Automatic Controls Conference,* Denver, Col., June 18-20, 1979, pp. 42-55.

[16]Meirovitch, L., *Computational Methods in Structural Dynamics,* Sijthoff-Noordhoff International Publishers, Alphen aan den Rijn, The Netherlands, 1980, Chs. 7 and 8.

[17]Meirovitch, L., "A New Method of Solution of the Eigenvalue Problem for Gyroscopic Systems," *AIAA Journal,* Vol. 12, Oct. 1974, pp. 1337-1342.

[18]Anderson, B.D.O. and Moore, J.B., *Linear Optimal Control,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1971, p. 99.

[19]Balas, M.J., "Enhanced Modal Control of Flexible Structures via Innovations Feedthrough," *Proceedings of the Second VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft,* Blacksburg, Va., June 1979, pp. 677-700.